# ENGINEERING MATHEMATICS

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## THE USE OF MATHEMATICAL MODELING TO DETERMINE THE NEED CONFIGURATION OF UNMANNED AIRCRAFT HAVING A VARIABLE STRUCTURE IN FLIGHT

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A balancer control in conjunction with the classical aerodynamic controls will significantly reduce the landing distance. We find the equations of motion both taking into account a movable load and without it, for unmanned aerial vehicle having a variable structure in flight.

*Keywords: differential equations of motion, variable structure, unmanned aerial vehicle.*

## Introduction

An availability of prepared landing site is a significant restriction at operation of unmanned aerial vehicles (UAVs). High maneuverability allows significantly extend the area of their application, such as the maneuver of landing on a limited site having a difficult surrounding terrain. These sites can be urban, forest or mountain ones.

Currently, the study to improve of maneuverability of aerial vehicle is carried out. An object of research is the aerial vehicle made by the normal scheme with a fixed-wing. The process of movement on the landing step is considered. In [1] this maneuver is described as pancakes landing. The device during such a maneuver "fails" to a certain height, and then there is an alignment and a touchdown. The load on the chassis at the moment of touching the ground is much, but it is possible to reach the point of landing at the lowest possible horizontal and vertical speed. Performing of the maneuver of parachute landing without significant damage to the aerial vehicle is carried out by highly qualified pilots.

In [2, 3] the term "perched landing" is adopted. It literally means "landing on the perch". The authors use it to indicate a landing onto a support so that vertical and horizontal movement velocity of the vehicle is zero, or close to it, at the moment of directly landing. A final step of the parachute landing maneuver, so called "a post stall" , is a damping of vertical and horizontal velocity by output of the device to critical angles of attack. This significantly increases the drag force and the lift of the wing falls, but steady movement in these modes is possible, as evidenced by a study published in [3, 4].

The mathematical representation of aerodynamic models in [2, 3, 4] describes the coefficient of aerodynamic drag force and the lift in the following form:

$$C_y = sin2\alpha,$$

$$C_x = |sin\alpha|.$$

Performing of the parachute landing maneuver, and especially its final part requires a large amount of control moment. For example, to this end an elevator on the device performed by the normal aerodynamic scheme is used in [3]. However, an efficiency of aerodynamic controls is not enough at speeds close to stall one. In this case, an application of a balancer control is possible. Its use was first described in [5]. The problem of research of the dynamics of motion of device with a balancer control, in the mechanics of the system is described as a movement of solid bodies. Differential equations of motion of solid bodies system in the annex to the motion of the autonomous transport systems consisting of two or more solid bodies were previously obtained in many papers, including [6, 7 and 8]. In the most general form such equation are described in [9, 10, 11, 12] and can be represented as follows:

$$\dot{q} = \tilde{H}\nu,$$

$$M(q)\dot{\nu} = f(t, q, \nu) + \tilde{H}^T(q)G^T(q, t)\lambda,$$

$$g(q, t) = 0,$$

where $q$ is a vector of generalized coordinates. It is used to determine a configuration of the system of several bodies at any time $t$; $M(q)$ is a symmetric positive definite matrix of weight; $f(t, q, \nu)$ is a function representing the contribution of the centrifugal force, the Coriolis force and the conditions of external influence; $H(q)$ is a matrix representing a kinematic relationship between the velocity variable $\upsilon$ and a derivative of the generalized coordinate $q$, expressed as: $\nu = H(q)\dot{q}$. Often $H$ is an identity map. In the case of spatial movement, a full representation is more convenient, because it allows to avoid the usual difficulties with the presentation of the features. $\tilde{H} = H^T(HH^T)^{-1}$; $g$ is a system of algebraic equations describing the limits. We assume that the function is twice continuously differentiable, $G$ is a Jacobian, $\lambda$ is a vector of Lagrange multipliers.

The movement of an aerial vehicle on the landing step on a site with difficult terrain boarding is described in [3, 6]. We denote it as a landing parachute trajectory. During reduction it is necessary to control the kinematic parameters characterizing the motion. They are an acceleration, angular velocity and the actual position coordinates. On the basis of their actual values one decides to continue a programm movement by the selected trajectory, or to return to the original trajectory, or to move to the closest alternative trajectory.

However, in the early steps of design to assess the behavior of a complex system it is advisable to to simulate the motion of the UAV with a variable in flight structure using the differential equations of motion of a solid [12]. The main difference of studied equations is that a moment of inertia is not constant, but depends on the time.

## 1. Mathematical formulation of the problem

A shift of load relative to the platform allows to change a balance of the aerial vehicle. This leads to the fact that one can control the process of its movement not only by the aerodynamic controls, but also by not compensated occurring forces and moments.

To change the relative position of a mass center and a pressure center one can move the individual apparatus units located inside the fuselage, as shown in Fig. 1, a), or to turn the load on a hinge, as shown in Fig. 1, b).

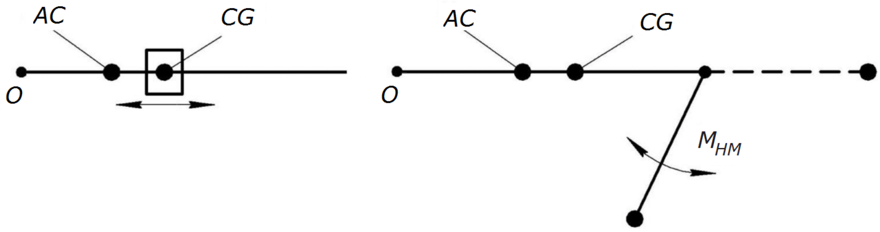


**Fig. 1**. Principle scheme of UAV control

Thus, the equations of UAV motion on the landing step taking into account the structural changes during the landing maneuver must contain the following parameters:

- weight of a platform and weight of a load;
- value of displacement of the gravity center of the load relative to the pressure center;
- value of displacement of a mass center of the load relative to its axis of rotation;
- value of removal of the load axis of rotation relative to a mass center of the platform;
- azimuth of an angle of load rotation.

Future we will consider the change of a device configuration by the scheme shown in Fig. 1, b). Let $\rho_{pl}$ be a radius of inertia platform, $m_{pl}$ and $m_l$ be a mass of load and platform, respectively. Let us present the moment of UAV inertia with variable structure in flight by the following expression, as well as by its derivative:

$$I_z = J_z + m_l b^2 + m_l l^2 \left( \frac{2b}{l} (\sin\phi + \cos\phi) + 2\sin\phi\cos\phi \right), \tag{1}$$

$$\dot{I}_z = 2m_l l^2 \left( \frac{b}{l} (\cos\phi - \sin\phi) + \left(\cos^2\phi - \sin^2\phi\right) \right). \tag{2}$$

In view of the equation of the aerial vehicle existence $m = m_{pl} + m_l$ and a mass ratio of the load and the platform $k_m = \frac{m_{pl}}{m_l}$, own moment of inertia of the platform is represented by the relation $J_z = m_{pl}\rho_{pl}^2$.

In view of the accepted expressions, (1) and (2) take the following form:

$$I_z = m\frac{1}{k_m+1} \left( k_m\rho_{pl}^2 + b^2 + l^2 \left( \frac{2b}{l} (\sin\phi + \cos\phi) + 2\sin\phi\cos\phi \right) \right), \tag{3}$$

$$\dot{I}_z = m\frac{l^2}{k_m+1} \left( \frac{2b}{l} (\cos\phi - \sin\phi) + 2\left(\cos^2\phi - \sin^2\phi\right) \right). \tag{4}$$

In view of the accepted expressions, a displacement of the gravity center relative to the mass center along axes takes the following form:

$$\begin{aligned} X_{mc} &= \frac{b + l\cos\phi}{k_m+1} = \frac{b}{k_m+1} + \frac{l}{k_m+1}\cos\phi, \\ Y_{mc} &= \frac{l}{k_m+1}\sin\phi. \end{aligned} \tag{5}$$

The differential equation of angular motion is

$$\frac{d\omega_z}{dt} = \frac{\dot{I}_z}{I_z}\omega_z + C_{m_z} q \frac{Sb_A}{I_z} - C_y q \frac{Sb_Y}{I_z}.$$

Let us rewrite it based on the obtained expressions

$$\begin{aligned} \frac{d\omega_z}{dt} = & \frac{l^2 \left( \frac{2b}{l} (\cos\phi - \sin\phi) + 2\left(\cos^2\phi - \sin^2\phi\right) \right)}{k_m \rho_{pl}^2 + b^2 + l^2 \left( \frac{2b}{l} (\sin\phi + \cos\phi) + 2\sin\phi\cos\phi \right)} \omega_z + \\ & + \frac{gq}{p} \frac{(k_m+1)\left( C_{m_z} b_A - C_y \left( b_{constY} + \frac{b}{k_m+1} + \frac{l}{k_m+1}\cos\phi \right) \right)}{\left( k_m \rho_{pl}^2 + b^2 + l^2 \left( \frac{2b}{l} (\sin\phi + \cos\phi) + 2\sin\phi\cos\phi \right) \right)}, \end{aligned} \tag{6}$$

where $p = \frac{G}{S} = \frac{mg}{S}$ is a specific wing load, $N/m^2$; $b_Y$ is a distance from UAV center of mass to its pressure center, where $b_Y = b_{constY} + b_{const\_b} + b_{per\_l}$. It contains a constant part $b_{constY}$, representing the mutual location of a center of gravity of platform and a pressure center, constant part $b_{const\_b}$, representing the load hinge displacement value, and a variable part $b_{per\_l}$, representing the size of the removal of the load position with respect to the hinge and its angular position.

Differential equations of translational motion of UAV mass center based on the specific load on the wing are

$$\frac{dV_x}{dt} = g\left( u - \frac{C_x q}{p} - \sin\theta \right) + \omega_z V_y, \tag{7}$$

$$\frac{dV_y}{dt} = g\left( \frac{C_y q}{p} - \cos\theta \right) - \omega_z V_x, \tag{8}$$

where $u = \frac{P}{G} = \frac{P}{mg}$ is a thrust-weight ratio.

The resulting system of differential equations of motions of UAV with a variable structure in flight, taking into account (6), (7) and (8), must be supplemented by the relations

$$\frac{dH}{dt} = V_x \sin\theta + V_y \cos\theta, \ \frac{dL}{dt} = V_x \cos\theta + V_y \sin\theta, \ \frac{d\theta}{dt} = \omega_z. \tag{9}$$

Based on [3], models of aerodynamic coefficients, a dynamic pressure and an angle of attack, as well as the dimensionless geometrical parameters can be presented in the following way:

$$C_x = C_{x_0} + \frac{C_y - C_{y_0}^2}{\pi\lambda} + C_x^{\delta_e} \cdot \delta_e, \tag{10}$$

$$C_y = C_{y_0} + C_y^{\alpha} \cdot \alpha + C_y^{\delta_e} \cdot \delta_e, \tag{11}$$

$$C_{m_z} = C_{m_{z_0}} + C_{m_z}^{\alpha} \cdot \alpha + C_{m_z}^{\delta e} \cdot \delta_e, \tag{12}$$

$$q = \frac{\rho V^2}{2} = \frac{\rho\left(V_x^2 + V_y^2\right)}{2}, \tag{13}$$

$$\alpha = \arctan\frac{V_y}{V_x} = \arcsin\frac{\sqrt{V_x^2 + V_y^2}}{V_y}, \tag{14}$$

$$\bar{b}=\frac{b}{l},\ \bar{b}_A=\frac{b_A}{l},\ \bar{b}_{constY}=\frac{b_{constY}}{l},\ \bar{\rho}_{pl}=\frac{\rho_{pl}}{l},\bar{b}_{constY1}=\frac{b_{constY}}{b_A},\ \bar{\rho}_{pl}=\frac{\rho_{pl}}{b_A}. \tag{15}$$

In (6), (7), (8) and (9) take into account the change of a distance between the aerodynamic center and the mass center, as described earlier by expressions (5), as well as accepted notations (10), (11), (12), (13) and (14). Represent the resulting system of differential equations in dimensionless form with respect to a congestion:

$$\begin{gathered}
\frac{1}{g}\frac{dV_x}{dt}=u-\frac{q}{p}\left(C_{x0}+\frac{C_{y0}\left(1-C_{y0}\right)+C_y^{\alpha}\alpha+C_y^{\delta e}\delta_e}{\pi\lambda}+C_x^{\delta e}\delta_e\right)-\sin\theta+\frac{\omega_z V_y}{g},\\
\frac{1}{g}\frac{dV_y}{dt}=\frac{q}{p}\left(C_{y0}+C_y^{\alpha}\alpha+C_y^{\delta e}\delta_e\right)-\cos\theta-\frac{\omega_z V_x}{g},\\
\frac{l}{g}\frac{d\omega_z}{dt}=\frac{q}{p}\frac{\left(C_{m_z0}+C_{m_z}^{\alpha}\alpha+C_{m_z}^{\delta e}\delta_e\right)\bar{b}_A\left(k_m+1\right)-}{k_m\bar{\rho}_{pl}^2+\bar{b}^2+2\left(\bar{b}\left(\sin\phi+\cos\phi\right)+\sin\phi\cos\phi\right)}\\
-\frac{q}{p}\frac{\left(C_{y0}+C_y^{\alpha}\alpha+C_y^{\delta e}\delta_e\right)\left(\bar{b}_{constY}\left(k_m+1\right)+\bar{b}+\cos\phi\right)}{k_m\bar{\rho}_{pl}^2+\bar{b}^2+2\left(\bar{b}\left(\sin\phi+\cos\phi\right)+\sin\phi\cos\phi\right)},\\
\frac{dH}{dt}=V_x\sin\theta+V_y\cos\theta,\ \frac{dL}{dt}=V_x\cos\theta+V_y\sin\theta,\ \frac{d\theta}{dt}=\omega_z.
\end{gathered} \tag{16}$$

Assume that the motion is performed so that an angle of attack remains constant and within the permissible (flight) values or slightly changed, then, without prejudice to the reliability of the described process, one can eliminate the terms in models of aerodynamics. Also assume that the speed on the glide path remains constant, then the dynamic pressure can be assumed to be constant. One can also ignore the values having the second order of smallness. So based on (16), the following system is obtained:

$$\begin{gathered}
\frac{1}{g}\frac{dV_x}{dt}=u-\frac{q}{p}\left(C_{x0}+\frac{C_{y0}+C_y^{\delta e}\delta_e}{\pi\lambda}+C_x^{\delta e}\delta_e\right)-\sin\theta+\tfrac{\omega_z V_y}{g},\\
\frac{1}{g}\frac{dV_y}{dt}=\frac{q}{p}\left(C_{y0}+C_y^{\delta e}\delta_e\right)-\cos\theta-\frac{\omega_z V_x}{g},\\
\frac{l}{g}\frac{d\omega_z}{dt}=\frac{q}{p}\bar{b}_A\left(k_m+1\right)\frac{\left(C_{m_z0}+C_{m_z}^{\delta e}\delta_e\right)-\left(C_{y0}+C_y^{\delta e}\delta_e\right)\left(\frac{\bar{b}_{constY}}{\bar{b}_A}+\frac{\bar{b}+\cos\phi}{\bar{b}_A\left(k_m+1\right)}\right)}{k_m\bar{\rho}_{pl}^2+\bar{b}^2+2\left(\bar{b}\left(\sin\phi+\cos\phi\right)+\sin\phi\cos\phi\right)},\\
\frac{dH}{dt}=V_x\sin\theta+V_y\cos\theta,\ \frac{dL}{dt}=V_x\cos\theta+V_y\sin\theta,\ \frac{d\theta}{dt}=\omega_z.
\end{gathered} \tag{17}$$

One can ignore a movable load, then system (17) takes the following form:

$$\begin{gathered}
\frac{1}{g}\frac{dV_x}{dt}=u-\frac{q}{p}\left(C_{x0}+\frac{C_{y0}}{\pi\lambda}\right)-\frac{q}{p}\left(\frac{C_y^{\delta e}}{\pi\lambda}+C_x^{\delta e}\right)\delta_e-\sin\theta+\frac{\omega_z V_y}{g},\\
\frac{1}{g}\frac{dV_y}{dt}=\frac{q}{p}C_{y0}+\frac{q}{p}C_y^{\delta e}\delta_e-\cos\theta-\frac{\omega_z V_x}{g},\\
\frac{b_A}{g}\frac{d\omega_z}{dt}=\frac{q}{p}\frac{1}{\bar{\rho}_{pl}^2}\left(C_{m_z0}-\bar{b}_{constY1}C_{y0}\right)+\frac{q}{p}\frac{1}{\bar{\rho}_{pl}^2}\left(C_{m_z}^{\delta e}-\bar{b}_{constY1}C_y^{\delta e}\right)\delta_e,\\
\frac{dH}{dt}=V_x\sin\theta+V_y\cos\theta,\ \frac{dL}{dt}=V_x\cos\theta+V_y\sin\theta,\ \frac{d\theta}{dt}=\omega_z.
\end{gathered} \tag{18}$$

## 2. Landing scheme

A transition in decline is carried out from horizontal flight, which is indicated in the diagram (see. Fig. 2) by points 1-2. Just as in the classic case, as already mentioned above, during horizontal flight an aerial vehicle is transferred in the landing configuration, the engine thrust, which is necessary for landing, is chosen, and, if it is necessary, the flaps are issued. The next step is a curved movement along a circular arc 2-3 with a radius A-2 to enter in the glide path 3-4. Its angle should be as large as possible, because at this step it is possible to significantly reduce the landing distance. In fact, this step is a dive. Out of a dive also occurs by a circular arc 4-5-6 with a radius B-5. Then there is little straight path 6-7, and again the movement is carried out on a circular path 7-8-9. If one follows to the proposed scheme, then in the neighborhood of point 9 a horizontal velocity is small and path 9-10 may be almost vertical line.

If we assume ideally this maneuver, it may be carried out as follows. Flight step 2-3-4-5 is performed with running engine. At this step one can control using aerodynamic controls (they are rudder, flaps and ailerons). After passing of point 5 the plane gains altitude, while the vertical velocity increases from zero to a certain value. If one uses just before accumulated kinetic energy to gain altitude, then vertical and horizontal velocities will be intensive decrease. This point corresponds to point 8. The pitch angle at the same time is approximately correspond to the angle of climb. A disruption of the flow on the wing is possible because a velocity falls. Aerodynamic controls at this time is not effective, but still it is possible to use a balancer control method. At the final step of landing it is necessary to ensure the landing of UAV with a zero angle of attack. It is possible by moving of a center of gravity.

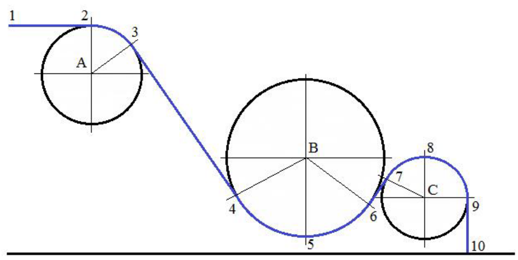


**Fig. 2**. Scheme of flight at the final step of landing

It is mentioned previously that the main resource for reducing of landing distance is the step 3-4. The dive is steeper, then the distance traveled is smaller. However, the dive must be carried out such that it is possible to get out of it and then climb. It must be remembered that a drawdown is possible. A drawdown increases with the velocity of dive and as a result with its steepness. The flight step 5-6 is represented by the arc of circle, which is equal to the arc of circle 4-5. This assumption is not entirely true, since the modes of flight, in which the radius of climb is less than the radius of the exit from the peak, are possible. Thus, at this step of flight the landing distance can be reduce. The radius of the arc 7-8-9 in the limit may be equal to zero. The transition from a positive pitch angle corresponding to climb, to zero pitch angle is carried out fully using a balancer control, because the flight velocity is close to zero and the aerodynamic controls do not work.

## 3. Determination of the experimental sample configuration

An experimental sample is developed and manufactured to verify the mathematical model. This sample is a remotely piloted fixed-wing aerial vehicle. The scheme to control the moving load, which is presented in Fig. 1, b), is realized in the design of the device.

Data of these configurations is shown in Table 1. We consider three variants of the ratio of movable load and platform masses, which are listed in the "Mass ratio" row of the table. For each variant of the mass ratio we set three positions of the movable load (see "Load position" row): F - forward position; N - neutral position; R - rear position. The values of displacement of movable load hinge are 90, 200, 300 and 400 mm. The values of displacement of the gravity center of load relative to its center of rotation are 185 and 260 mm. The actual position of the load in the different positions is represented by the coordinate $y$, which depends on the combination of parameters described above. The origin is in the toe of the root of the chord profile device. The actual distance is also shown in the table and is denoted by $left| \ Deltay \ right|$. For each condition a value of the main central moment of device inertia $J_y$ and its increment relative to the neutral state $J_y$ are calculated. The ratio of the value of device inertia moment in the front and rear positions to its value in the neutral position $k = \frac{J_y^{FR}}{J_y^N}$ is of practical interest.

**Table 1**

Configuration of the experimental sample

| Mass ratio | 5% | | | 13% | | | 25% | | |
|---|---|---|---|---|---|---|---|---|---|
| Load position | F | N | R | F | N | R | F | N | R |
| $y$, m | 0,440 | 0,434 | 0,429 | 0,458 | 0,443 | 0,428 | 0,486 | 0,456 | 0,426 |
| $\lvert\Delta y\rvert$, mm | 6 | - | 5 | 15 | | 15 | 30 | | 30 |
| $J_y, kg \cdot m^2$ | 4,735 | 4,715 | 4,699 | 4,817 | 4,763 | 4,718 | 4,957 | 4,842 | 4,750 |
| $\lvert\Delta J_y\rvert, kg \cdot m^2$ | 0,020 | - | 0,016 | 0,054 | | 0,045 | 0,115 | | 0,092 |
| $k = \frac{J_y^{FR}}{J_y^N}$ | 1,004 | - | 0,997 | 1,011 | | 0,991 | 1,024 | | 0,981 |
| $y$, m | 0,430 | 0,424 | 0,419 | 0,431 | 0,416 | 0,401 | 0,433 | 0,403 | 0,372 |
| $\lvert\Delta y\rvert$, mm | 6 | | 5 | 15 | | 15 | 30 | | 31 |
| $J_y \ ,kg \cdot m^2$ | 4,699 | 4,715 | 4,734 | 4,718 | 4,762 | 4,815 | 4,750 | 4,838 | 4,951 |
| $\lvert\Delta J_y\rvert, kg \cdot m^2$ | 0,016 | | 0,019 | 0,044 | | 0,053 | 0,088 | | 0,113 |
| $k = \frac{J_y^{FR}}{J_y^N}$ | 0,997 | | 1,004 | 0,991 | | 1,011 | 0,982 | | 1,023 |
| $y$, m | 0,442 | 0,434 | 0,426 | 0,464 | 0,443 | 0,422 | 0,499 | 0,456 | 0,414 |
| $\lvert\Delta y\rvert$, mm | 8 | | 8 | 21 | | 21 | 43 | | 42 |
| $J_y, kg \cdot m^2$ | 4,763 | 4,736 | 4,713 | 4,897 | 4,820 | 4,757 | 5,119 | 4,955 | 4,828 |
| $\lvert\Delta J_y\rvert, kg \cdot m^2$ | 0,027 | | 0,023 | 0,077 | | 0,063 | 0,164 | | 0,127 |
| $y$, m | 0,428 | 0,420 | 0,412 | 0,425 | 0,403 | 0,382 | 0,420 | 0,377 | 0,336 |
| $\lvert\Delta y\rvert$, mm | 8 | | 8 | 22 | | 21 | 43 | | 41 |
| $J_y, kg \cdot m^2$ | 4,712 | 4,758 | 4,808 | 4,752 | 4,879 | 5,020 | 4,818 | 5,076 | 5,370 |
| $\lvert\Delta J_y\rvert, kg \cdot m^2$ | 0,046 | | 0,050 | 0,127 | | 0,141 | 0,258 | | 0,294 |
| $k = \frac{J_y^{FR}}{J_y^N}$ | 0,990 | | 1,011 | 0,974 | | 1,029 | 0,949 | | 1,058 |

Configuration points of the experimental sample are shown in the graph, see Fig. 2.

The $x$-axis shows the relative weight of the load, %. Load weight refers to the weight of UAV, which is not equipped with a balance weight. The $y$-axis shows the values of the relative inertia moments. The inertia moment of UAV in three possible balance weight position (forward, back and neutral displacement) is related to the inertia moment of UAV, which is not equipped with a balance weight.

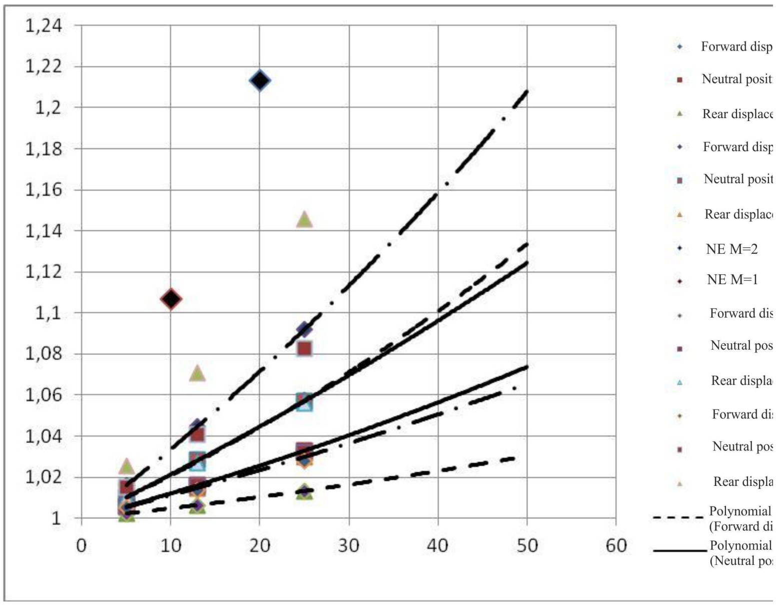


**Fig. 3**. To determine the load mass required for detection of the effect

Based on the presented values it is easy to see that in order the investigated effect be confirm experimentally, the values of relative inertia moment should be in the range of 1.11 to 1.21 relative units of inertia moment. This range corresponds to the numerical experiment conducted earlier.

For the current structure of UAV and the given method to change an inertia moment of device it is necessary that the following conditions are fulfilled.

1. load weight is 5 kg;
2. value of displacement of the load suspension point with respect to the hinge is 260 mm;
3. hinge offset is 400 mm.

## Summary

Suppose that the result of the work demonstrates that the reduction of the device from a height of 30 m at a distance less than 200 meters is possible. Then one can conclude the following. Firstly, the reduction for the glide path such that its angle is greater than $4^\circ$ is possible to perform. Secondly, the use of balancer control at all steps of decline, especially in the final one, will further reduce the landing distance.

It is advisable to use a balancer control at the time of movement about point 2. Moreover, if a static stability of device is reduced, then more intense reversal is possible. Note that in the limit a turning radius may be equal to zero. During the dive, it is desirable do not to reduce a static stability, due to the fact that, as was shown in Ostoslavsky paper [13], it complicates the conclusion of the aerial vehicle from the dive.

Because the proposed trajectory is decline, it is necessary to consider the deviation of the actual trajectory from the given one as a control optimization criterion. Final functional takes the form

$$J = \frac{1}{2}\left(\lambda_1 L\left(T\right)^2 + \lambda_2 V_x\left(T\right)^2 + \lambda_3 V_y\left(T\right)^2 + \lambda_4 a_y\left(T\right)^2\right) +$$
$$+\frac{1}{2}\int_0^T \left\{\lambda_5\left(H_0 f_H\left(t\right) - H\left(t\right)\right)^2 + \lambda_6\left(f_L\left(t\right) - L\left(t\right)\right)^2 + \lambda_7\left(\delta_e\right)^2 + \lambda_8\left(\phi\right)^2\right\} dt$$

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## ИСПОЛЬЗОВАНИЕ МАТЕМАТИЧЕСКОГО МОДЕЛИРОВАНИЯ ДЛЯ ОПРЕДЕЛЕНИЕ ПОТРЕБНОЙ КОНФИГУРАЦИИ БЕСПИЛОТНОГО ЛЕТАТЕЛЬНОГО АППАРАТА С ИЗМЕНЯЕМОЙ В ПОЛЕТЕ СТРУКТУРОЙ

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Балансирное управление совместно с классическим аэродинамическими органами управления позволит существенно сократить дистанцию посадки. Выводятся уравнения движения с учетом подвижного груза и без него, для беспилотного летательного аппарата с изменяемой в полете структурой.

*Ключевые слова: дифференциальные уравнения движения, изменяемая структура, беспилотный летательный аппарат.*

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