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CALCULATION OF SCATTERING ELECTROMAGNETIC FIELD OF BODY WITH RECTANGULAR SECTION

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A calculation of scattered electromagnetic field, under the condition of a plane wave is incident on the body having a rectangle cross section, is carried out. Using an approximation of the physical optics, we obtain the formulas for calculation of the scattered field, which can be used for engineering evaluation.

Keywords: scattered field; physical optics; plane wave is incident.

Introduction

In order to install base stations antennas on buildings roofs, aircraft landing systems on aerodrome, wireless internet antennas, one need to take into account an impact of local objects on the radiation field. As a rule, obstacles for the base station antennas are made by air conduit. Buildings can affect on the radiation field, if the base station is located within a dense housing. Supporting columns can affect on the field for wireless internet antennas installed inside of the building.

In order to install the antenna, it is necessary to calculate the impact of local objects on the field and to evaluate the obstacles, which can be made by them.

The scattered field can be calculated using the physical theory of diffraction (PTD), which is a high-frequency asymptotic theory. Using this theory, we can solve the problem of electromagnetic waves scattering on complex shape bodies [1]. A disadvantage of this theory is the fact that the tool for calculating the scattered field is very difficult and includes rather large mathematics sections and specific mathematical methods, such as the theory of functions of complex variable, the pass method [2], the asymptotic methods [3].

A precision of the scattered field calculation, conducted by rigorous PTD methods, is not required for engineering calculation. The estimated and scattered field values are enough in order to either to select new location for the antenna installation or to make recommendations how to reduce the impact of local objects on the scattering field.

1. Problem Formulation

Consider a diffraction of plane electromagnetic wave, which is incident on parallelepiped with rectangle cross-section, using the physical optics (PD) approximation. In order to simplify the calculation, we use only uniform current components [1].

Consider an incident electromagnetic wave. The wave excites the same scattering sources at parallelepiped faces. Assume that the parallelepiped surface is perfectly conductive.

The plane wave is incident on the body at angle of φ_0.The problem is solved for the plane case, that is we consider only two-dimensional case. The obtained solution can always be extended to the three-dimensional case [4]. We define the coordinate system so that x and y axis pass through the rectangle diagonal, see Fig. 1. Let a and b be a width and a length of the rectangle, respectively.

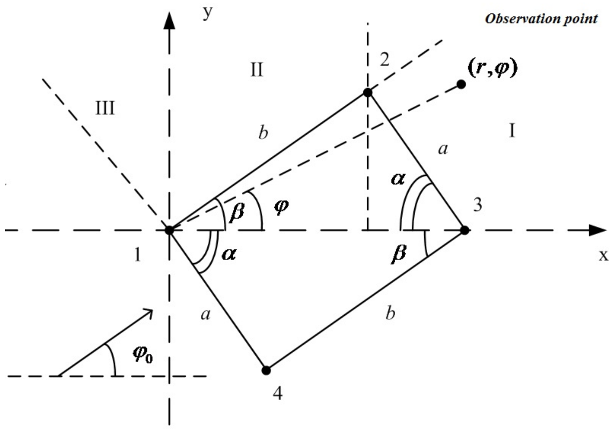

Fig. 1. Geometry of rectangle field scattering problem

Denote the first and the fourth quadrant angle between the x axis and the side of the rectangle by β and α, respectively. In this case, coordinates of rectangle vertices are

$$1(0;0), 2\left(b\cdot\cos\beta,\quad b\cdot\sin\beta\right), 3\left(a\cdot\cos\alpha, b\cdot\cos\beta\right), 4\left(a\cdot\cos\alpha,\ -a\cdot\sin\alpha\right)$$

We avoid the traditional method of integration to calculate the scattered field in the approximation of physical optics, because the method involves difficult calculations and use of the saddle-point method, which greatly complicate the problem.

In order to simplify the calculation we use the solution obtained for a tape, which is given in [1]. In this case, the body with the rectangular section, which is incident of plane electromagnetic wave, can be represented as a combination of four tapes $1-2$, $2-3$, $3-4$, $4-1$, each tape is a face of the body and calculate the field scattered by each face.

The field scattered by tape consists of edge waves, which are determined by the expression

$$u_s^{(0)} = u_0\Phi_s^{(0)}\left(\varphi,\varphi_0\right)\frac{\exp\left(i\left(kr+\pi/4\right)\right)}{\sqrt{2\pi kr}},$$

where
u_0 is an amplitude of the incident wave;
$\Phi_s^{(0)}\left(\varphi,\varphi_0\right) = \sum_{i=1}^{N} f^{(0)}(i)e^{-i\psi_i}$ is a function describing an effect of a uniform edge wave component on the scattered field;
$k = \frac{2\pi}{\lambda}$ is a wave number;
r is a distance between the center of coordinates and the observation point;
φ_0 is an angle of the incident wave direction, $0 \leq \varphi_0 \leq \pi$.

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2. Problem Solving Techniques

Note that the problem is symmetric. Therefore we consider the scattered field in $0 \leq \varphi \leq \pi$.

We represent domain $0 \leq \varphi \leq \pi$ as a set of sectors. Denote domain $0 \leq \varphi \leq \beta$ as sector I; domain $\beta < \varphi \leq \frac{\pi}{2}$ as sector II; domain $\frac{\pi}{2} < \varphi \leq \pi - \alpha$ as sector III; domain $\pi - \alpha < \varphi \leq \pi$ as sector IV.

Consider each sector individually and find an expression for the function $\Phi_s^{(0)}(\varphi, \varphi_0)$ and a contribution of each edge wave of body edges.

In domain $0 \leq \varphi \leq \pi$ the field is represented by a sum of 3 edge waves from 2, 3 and 4 edges. The scattered field can be represented as the sum

$$\Phi_S^{(0)} = f_2^{(0)} e^{i\psi_2} + f_3^{(0)} e^{i\psi_3} + f_4^{(0)} e^{i\psi_4}.$$

In domain $\beta < \varphi \leq \frac{\pi}{2}$ the field is represented by a sum of 3 edge waves from 1, 2 and 3 edges. The scattered field can be represented as the sum

$$\Phi_S^{(0)} = f_1^{(0)} e^{i\psi_1} + f_2^{(0)} e^{i\psi_2} + f_3^{(0)} e^{i\psi_3}.$$

In domain $\frac{\pi}{2} < \varphi \leq \pi - \alpha$ the field is represented by a sum of 2 edge waves from 1 and 2 edges. The scattered field can be represented as the sum

$$\Phi_S^{(0)} = f_1^{(0)} e^{i\psi_1} + f_2^{(0)} e^{i\psi_2}.$$

In domain $\pi - \alpha < \varphi \leq \pi$ the field is represented by a sum of 3 edge waves from 4, 1 and 2 edges. The scattered field can be represented as the sum

$$\Phi_S^{(0)} = f_1^{(0)} e^{i\psi_1} + f_2^{(0)} e^{i\psi_2} + f_4^{(0)} e^{i\psi_4}.$$

To calculate we need to substitute the local coordinates $(r_{1,\ 2,\ 3,\ 4},\ \ \varphi_{1,\ 2,\ 3,4})$ by the basic coordinates $(r,\ \varphi)$.

Let us define functions $f_1^{(0)}$, $f_2^{(0)}$, $f_3^{(0)}$ and $f_4^{(0)}$ for each domain. Consider domain $0 \leq \varphi \leq \beta$, see Fig. 2.

For edge 2, angles φ_{02} and φ_2 are

$$\begin{cases} \varphi_2 = \varphi + \beta \\ \varphi_{02} = \pi + \beta + \varphi_0 \end{cases}$$

Then function $f_2^{(0)}$ is

$$f_2^{(0)} = \frac{\sin \varphi_{02}}{\cos \varphi_{02} + \cos \varphi_2} = \frac{\sin(\pi + \beta + \varphi_0)}{\cos(\pi + \beta + \varphi_0) + \cos(\varphi + \beta)} =$$

$$= \frac{\sin(\beta + \varphi_0)}{\cos(\beta + \varphi_0) - \cos(\varphi + \beta)}$$

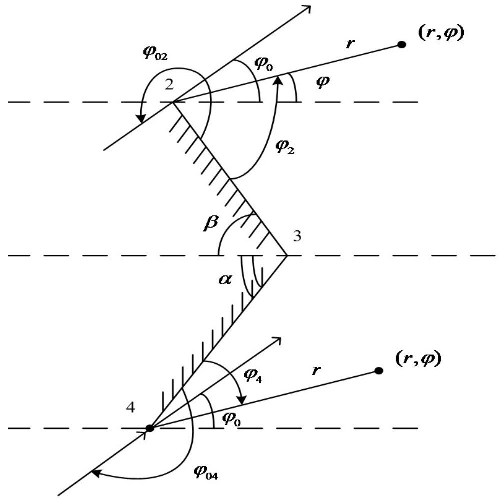

Fig. 2. Local coordinates for domain $0 \leq \varphi \leq \beta$

For edge 4, angles φ_{04} and φ_4 are

$$\begin{cases} \varphi_4 = \alpha - \varphi \\ \varphi_{04} = \pi + \alpha - \varphi_0 \end{cases}$$

Then function $f_4^{(0)}$ is

$$f_4^{(0)} = \frac{\sin \varphi_{04}}{\cos \varphi_{04} + \cos \varphi_4} = \frac{\sin(\pi + \alpha - \varphi_0)}{\cos(\pi + \alpha - \varphi_0) + \cos(\alpha - \varphi)} =$$

$$= \frac{\sin(\alpha - \varphi_0)}{\cos(\alpha - \varphi_0) - \cos(\alpha - \varphi)}$$

For edge 3, function $f_3^{(0)}$ is

$$f_3^{(0)} = -f_2^{(0)} - f_4^{(0)}$$

Then function $\Phi_S^{(0)}$ in domain $0 \leq \varphi \leq \beta$ can be written as

$$\Phi_S^{(0)} = f_2^{(0)} e^{i\psi_2} + f_3^{(0)} e^{i\psi_3} + f_4^{(0)} e^{i\psi_4},$$

where

$$\begin{cases} \psi_2 = -2k\,(b \cdot \cos\beta \cdot \cos\varphi + b \cdot \sin\beta \cdot \sin\varphi) \\ \psi_3 = -2k\,(a \cdot \cos\alpha + b \cdot \cos\beta) \cdot \cos\varphi \\ \psi_4 = -2k\,(a \cdot \cos\alpha \cdot \cos\varphi - b \cdot \sin\alpha \cdot \sin\varphi) \end{cases}$$

Similarly, we find function $\Phi_S^{(0)}$ for other sectors.

In domain $\beta < \varphi \leq \frac{\pi}{2}$ function $\Phi_S^{(0)}$ is

$$\Phi_S^{(0)} = f_1^{(0)} e^{i\psi_1} + f_2^{(0)} e^{i\psi_2} + f_3^{(0)} e^{i\psi_3}$$

$$f_1^{(0)} = \frac{\sin(\varphi - \beta)}{\cos(\varphi_0 - \beta) + \cos(\varphi - \beta)}$$

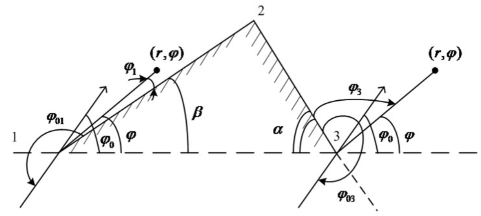

Fig. 3. Local coordinates for domain $\beta < \varphi < \frac{\pi}{2}$

$$f_3^{(0)} = \frac{\sin(\varphi_0 + \alpha)}{\cos(\varphi + \alpha) + \cos(\varphi_0 + \alpha)}$$

$$f_2^{(0)} = -f_1^{(0)} - f_3^{(0)}$$

$$\begin{cases} \psi_1 = 0 \\ \psi_2 = -2k\,(b \cdot \cos\beta \cdot \cos\varphi + b \cdot \sin\beta \cdot \sin\varphi) \\ \psi_3 = -2k\,(a \cdot \cos\alpha + b \cdot \cos\beta) \cdot \cos\varphi \end{cases}$$

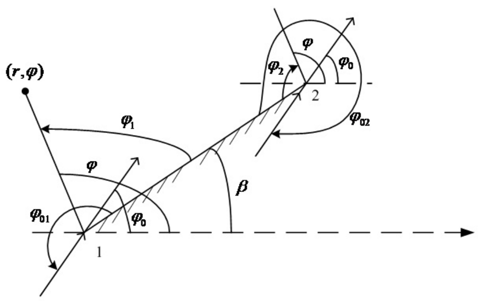

Fig. 4. Local coordinates for domain $\beta < \varphi < \frac{\pi}{2}$

In domain $\frac{\pi}{2} < \varphi \leq \pi - \alpha$ function $\Phi_S^{(0)}$ is

$$\Phi_S^{(0)} = f_1^{(0)} e^{i\psi_1} + f_2^{(0)} e^{i\psi_2}$$

$$f_1^{(0)} = \frac{\sin(\varphi_0 - \beta)}{\cos(\varphi_0 - \beta) - \cos(\varphi - \beta)}$$

$$f_2^{(0)} = \frac{\sin(\varphi_0 + \alpha)}{\cos(\varphi + \alpha) + \cos(\varphi_0 + \alpha)}$$

$$\begin{cases} \psi_1 = 0 \\ \psi_2 = -2k\,(b \cdot \cos\beta \cdot \cos\varphi + b \cdot \sin\beta \cdot \sin\varphi) \end{cases}$$

In domain $\pi - \alpha < \varphi \leq \pi$ function $\Phi_S^{(0)}$ is

$$\Phi_S^{(0)} = f_1^{(0)} e^{i\psi_1} + f_2^{(0)} e^{i\psi_2} + f_4^{(0)} e^{i\psi_4}$$

$$f_2^{(0)} = \frac{\sin(\varphi_0 - \beta)}{\cos(\varphi - \beta) - \cos(\varphi_0 - \beta)}$$

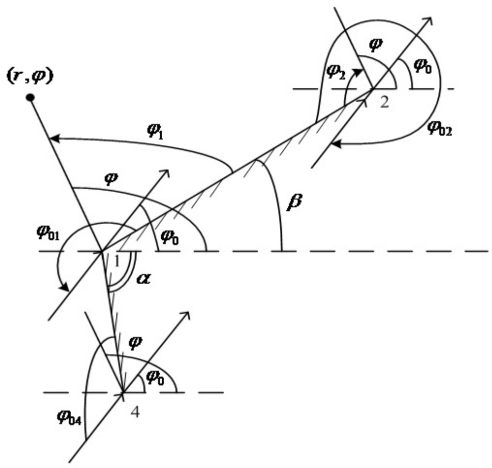

Fig. 5. Local coordinates for domain $\pi - \alpha < \varphi \leq \pi$

$$f_4^{(0)} = \frac{\sin(\varphi_0 + \alpha)}{\cos(\varphi_0 + \alpha) - \cos(\varphi + \alpha)}$$

$$f_1^{(0)} = -f_2^{(0)} - f_4^{(0)}$$

$$\begin{cases} \psi_1 = 0 \\ \psi_2 = -2k\,(b \cdot \cos\beta \cdot \cos\varphi + b \cdot \sin\beta \cdot \sin\varphi) \\ \psi_4 = -2k\,(a \cdot \cos\alpha \cdot \cos\varphi - b \cdot \sin\alpha \cdot \sin\varphi) \end{cases}$$

3. Obtained Results

For example, consider the particular case $\varphi_0 = 0$, $\alpha = 60°$, $a = 2\lambda$, $r = 100\lambda$. The result is presented in Fig. 6.

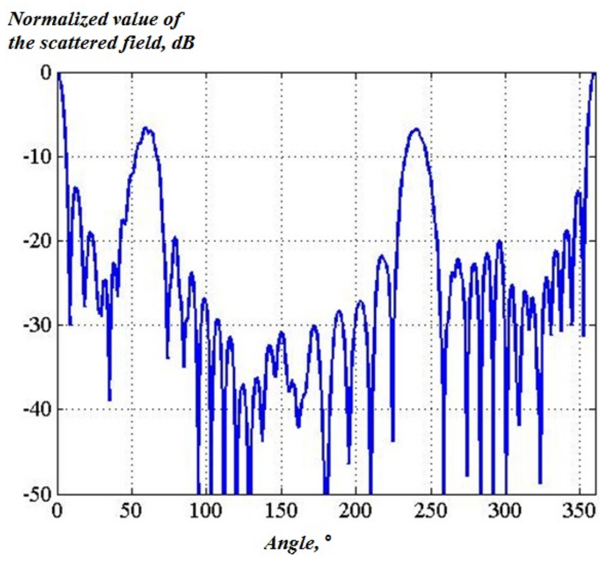

Fig. 6. A dependence of the normalized value of the scattered field on the angle

Fig. 6 shows that the maximum of the scattered field is directed as the incident wave. There are maximums at angles $\varphi = 60°$ and $\varphi = 240°$, because these angles correspond to rectangular edges 2 and 4. There is minimum at angle $\varphi = 180°$, because edge 3 is is on the shady side and has no significant effect on the scattered field.

Conclusions

We obtain formulas to calculate the scattered field of a rectangular body, using the approximation of physical optics and solving the problem of scattering of a plane electromagnetic wave on the tape. An example of the calculation for the particular case of the incident plane wave is presented.

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ВЫЧИСЛЕНИЕ РАССЕЯННОГО ЭЛЕКТРОМАГНИТНОГО ПОЛЯ ОТ ТЕЛА С ПРЯМОУГОЛЬНЫМ СЕЧЕНИЕМ

Д.С. Клыгач

В статье рассматривается расчет рассеянного электромагнитного поля при падении плоской волны на тело с поперечным сечением в виде прямоугольника. С помощью приближения физической оптики, получены формулы для расчета рассеянного поля, которые можно использовать для инженерной оценки.

Ключевые слова: рассеянное поле; физическая оптика; падение плоской волны.

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