

MATHEMATICAL MODEL OF 3D MAPS AND DESIGN OF INFORMATION SYSTEM FOR ITS CONTROL

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This article describes how to create an information system for storing 3D models of architectural complexes and urban districts on the example of the pre-war Stalingrad. The information system allows you to manage files, models, photos and video materials, plans and maps of urban neighbourhoods, the text information of a historical nature. Also it allows to place 3D model on the plans and maps circuits connecting the geographical coordinates from the model geometry. For data management information system it was started to develop web-based interface to the PostgreSQL database server and the Express development environment based NodeJS platform. The paper describes the mathematical model of the binding three-dimensional models to geographic coordinates related to the coordinate conversion based on the scale of the map data and the resolution of the image file with her image.

Keywords: geometric model; schema of database; ER diagram; planar disconnection; planar partition; PostgreSQL; NodeJS.

1. The Production of Design Problem

In early 1942 the leadership of Wehrmacht was decided to begin the year's biggest attack on the southern flank of the Eastern Front to destroy the group of Soviet forces west of the Don, and then grab the oil fields of the North Caucasus. In May 1942, Hitler ordered Army Group «B» consisting of the 6th Army of Friedrich Paulus and 4 Panzer Army Gotha, to move east towards the Volga and Stalingrad.

23 August 1942 forces of the 4th Air Force Luftwaffe produced the longest and the destructive bombardment of the city. Under the blows of German aircraft more than 90,000 civilians were killed and more than half of pre-war housing stock Stalingrad were destroyed. The city has been turned into a vast territory covered with burning ruins. The situation is exacerbated by the fact that after high-explosive bombs German bombers dropped incendiary bombs. A huge fiery vortex that burned down the central part of the city and spread to other areas of Stalingrad.

The devastation caused by Stalingrad were not uniform by areas. So Yermansky, Dzerzhinsky and Traktorozavodsky areas, which within 6 months held the front line of street fighting, were destroyed almost completely. Kirovsky area, escaped the German occupation, suffered serious destruction from shelling and air bombardment. Among the victims were the least Beketovka village who happened to be in the direction of the main

attacks on both sides. It became the center of Stalingrad on the 1943 - 1945 years old, there were placed all urban services, coordinating the restoration of the city.

We are making an attempt to recover some of the architectural structures of the city in 3D-format from the available photos and video-materials. Our attention to this problem, in particular, due to the fact that now the three-dimensional models are becoming important and almost indispensable tool for any management system designed for a wide range of practical problems [1], [2]. In this case, the entire set of three-dimensional models of architectural structures can be used for 3D-reconstruction of the different areas of the city-hero. This is supposed to create a centralized repository of 3D models and all associated files. To this end, we aim to create an appropriate information system, perform the following functions:

- loading and storage on the server files with 3D models of architectural structures,
- loading and storage of photos and video,
- loading and storage of image maps / plans of city districts
- registration and authentication for users of the system,
- Placement of three-dimensional models for the maps and plans of the city-hero » « binding three-dimensional models of objects preserved city plans
- loading and storage on the server text information related to a particular construction, historical events associated with it.

To solve this problem we developed ER-diagram of the database schema, which is implemented on the basis of a physical model of the database in the PostgreSQL database. Also it was built a mathematical model for solving the problem of accommodation « » three-dimensional models on the base surface in order to create an appropriate three-dimensional scenes, it was designed and implemented user interface software package, as well as built UML class diagram that displays the subject area and the interaction of the individual components of the software package.

Let us turn to the definition of concepts and notations required to build a mathematical model of 3D maps. We call by geometric model any pair of concerted sets (P, F) . Here $P \subset \mathbb{R}^3$ is a finite set of points in \mathbb{R}^3 , and $F \subset \cup Q^k, Q = \{1, 2, \dots, N\}, N = |P|, |A|$ denotes cardinality of A . In other words, the geometrical model is determined by its set of vertices of P and a set F of sequences of natural numbers $(n_{i1}, n_{i2}, \dots, n_{ik_i})$, such that i -th face of the geometric model is constructed from vertices with numbers $n_{i1}, n_{i2}, \dots, n_{ik_i} \in Q$. In this sense, we understand the consistency of P and F . For example, the geometric model of unit cube is (P, F) , where $P = \{(0, 0, 0), (1, 0, 0), (1, 1, 0), (0, 1, 0), (0, 0, 1), (1, 0, 1), (1, 1, 1), (0, 1, 1)\}$, $F = \{(1, 2, 3, 4), (1, 2, 6, 5), (2, 3, 7, 6), (3, 4, 8, 7), (4, 1, 5, 8), (5, 6, 7, 8)\}$. Note that the problem of constructing 3D models for photographic materials reminiscent of inverse problems of descriptive geometry. With some methods and algorithms for solving inverse problems of descriptive geometry can be found in the papers [3], [4]. However, in our case, these tasks have their own characteristics, in particular, consist in the fact that the required calculations of geometrical characteristics of real objects in the substantial absence of any drawings. In this regard, a number of specific inverse problems of computational geometry for the reconstruction of a 3D model.

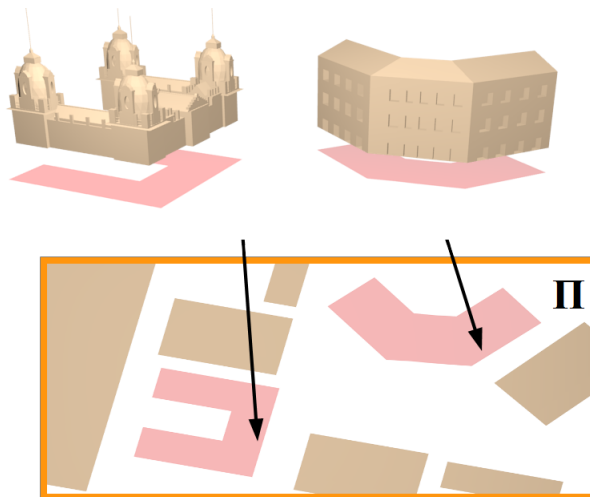


Fig. 1. The binding model to map/plan

Further we consider the rectangle $\Pi = \{(x, y) : 0 \leq x \leq A, 0 \leq y \leq B\}$. We assume this rectangle is a model of geographic map or plan of the city parts or any areas. Let $M = \{M_j, j = 1, \dots, m\}$ be a planar partition Π – complex simple polygons $M_j \subset \Pi$, which do not intersect at interior points and thus $\Pi = \cup_j M_j$. By X we denote a set of models. If we consider the subset of models $Y \subset X$ a particular scene, the bijection $\varphi : Y \subset X \rightarrow M$ binds to the appropriate model polygon on the plane Π (see. picture. 1). The mapping φ is the mathematical content of «one-to-one» relationship models and map_objects on in the following sections of the database schema. The main problem of the problem of locating the model map is calculated vertex coordinates of model in accordance with the coordinates of the polygon vertices M_j , wherein the coordinates of the vertices of the polygon in the set requires some global coordinate system. As such a coordinate system in our work selected geographic coordinates - latitude and longitude points on the ground.

2. Database Schema

Here is the description of the elements of ER-database diagrams.

«Models» table contains information on the three-dimensional model of the specified object: file name, a description of the model, as well as a marker showing whether the model is generated in the program or reconstituted as photo documents. Table «models» associated with table «map_objects», storing a reference to a KML file with coordinates, an identifier, and additional information about the real object on a map - recording in «geomaps» table. All models can be integrated into an architectural complex (Table «arch_complexes»). This table lists the model IDs of the models of the table and a description of the complex. In the three-dimensional scene can display various architectural complexes, for example, the central square and surrounding streets, the building of the station and the station square, etc. Therefore, the parent table «scenes» contain links to the child table «arch_complexes». Table allows you to save available photos of objects for further visualization. Summary table for card objects (table «map_objects»)



Fig. 2. An example of the map image file (plan) of the Plant «Krasny Oktyabr» in Stalingrad (September 1942)

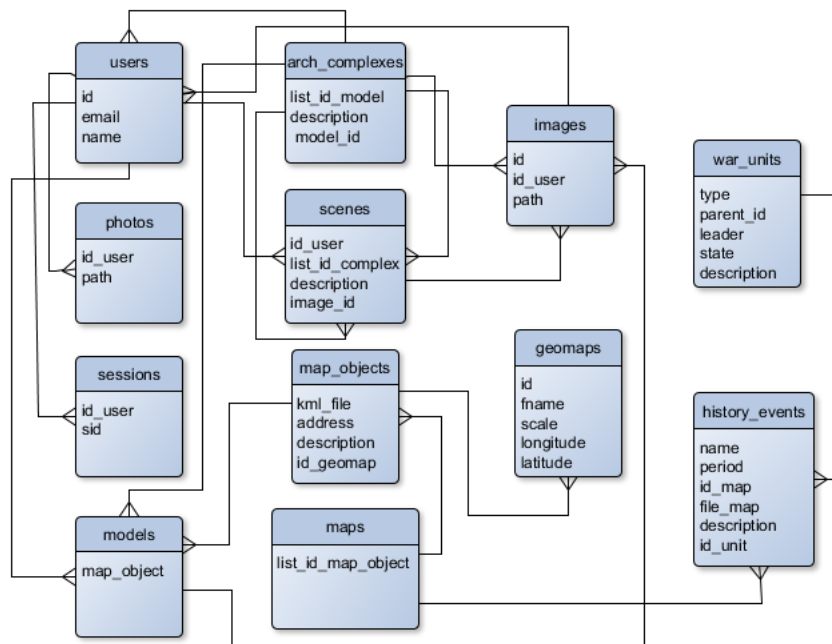


Fig. 3. ER-diagram of a database schema

contains a reference to the ID card «geomaps», to which is attached the object. Table «history_events» includes the necessary historical information about the period, the place and the military units participating in the displayed three-dimensional scene. The types and numbers of military units, their commanders and other related information is stored in the table «war_units». Auxiliary tables «users», «sessions» are required for registration in the system development models, as well as to save the current connection settings.

The database schema is implemented in SQL DBMS PostgreSQL. Selecting the database to the open source community due to the extensive capabilities of the relational database management system in the development of projects that manipulate large volumes of data, as well as its object classes.

3. Coordination of Geographical and Pixel Coordinates

For site management maps and plans of the city areas we was designed web interface that allows:

- to download from the data store file with the image of the plan/map,
- to display the file objects previously stored with the image of the plan/map,
- to build interactively a polygonal area on the plan/map, linking it with the 3D model, which is the assembly of the 3D scene is set in place the polygon (object map/plan).

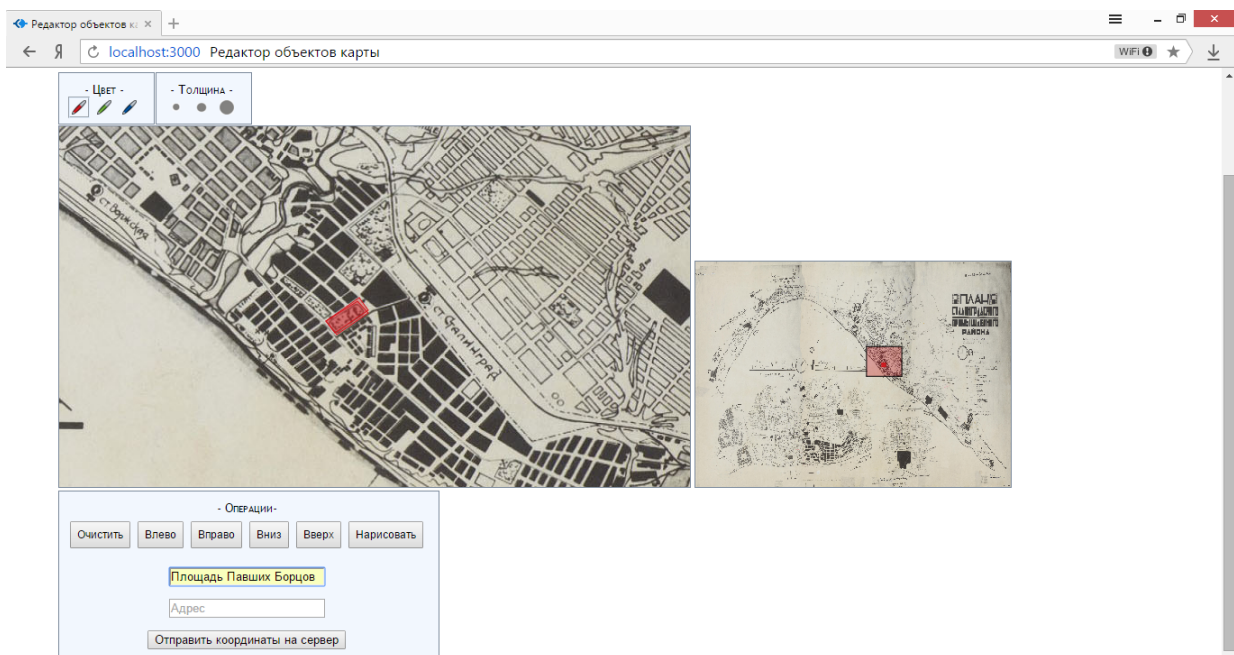


Fig. 4. Editor the map objects

In this system, we give the opportunity to construct a map object (map_object) highlighting the polygonal area in the image loaded map/plan to user. This functionality is based on the browser's handling of events and the property type <canvas>. Since the event object allows to get a «mouse» cursor coordinates in pixels, it is natural there is a problem of translation of geographical coordinates in the pixel and back. To this end loaded in the map database retained its special characteristics:

- the geographical coordinates of A_0, L_0 upper-left corner of the map/plan,

- coefficient λ of scale of the map: the proportion of degrees per one pixel of the map. This value can be calculated by the following formula

$$\lambda = \frac{m}{\mu p},$$

where p is a number of pixels per one centimeter in the original image map, $\mu = 10^8/9$ and m is a integer number included in the standard scale indication, for example, a map with the scale of 1 : 50,000, the value of $m = 50000$.

Let X, Y be a pixel coordinates of a point on the map display scale with parameter λ . Then, this point corresponds to a point on the ground with the longitude of L and altitude of A , which can be calculated by the formula

$$\begin{cases} L = L_0 + X\lambda, \\ A = A_0 - Y\lambda \end{cases}.$$

The polygonal area allocated by the user on the map given by a list of pairs of geographical coordinates of the vertices of the form $(L_i, A_i) i = 1, \dots, k$.

4. Placing 3D Models on a Planar Partition

Under the simulation of architectural structures it was used local coordinate system - usually in a 3D model editor in the modelling process are placed in a certain neighbourhood of the origin. When you export the model in any format the coordinates of the vertices it will written in the file in this coordinate system. However, the real object is located in the global coordinate system - in this case, the geographical coordinates. When choosing a binding model of the real object to the location on the map in the database recorded the geographic coordinates of the corresponding polygonal area. Therefore, it is necessary to develop a method for transforming 3D coordinates of the vertices of the model in geographic coordinates. This is the mathematical basis of placing 3D models on a map. In our paper, we propose the following method:

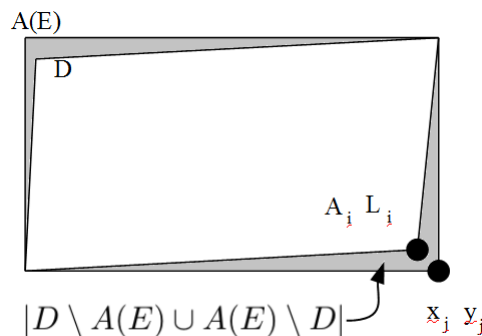


Fig. 5. Minimizing the area $|D \setminus A(E) \cup A(E) \setminus D|$

- From the database it is got map object that is «attached» with processed model.

- The object map defines the geographical coordinates $(L_i, A_i) i = 1, \dots, k$ of polygonal 3D model layout area. We denote the corresponding polygonal area through D . item Based on the coordinates of the vertices of the model we calculate its projection onto the plane $z = 0$. Projection is a certain polygon E , predetermined points with coordinates $(x_j, y_j), j = 1, \dots, p$. item Find affine transformation $A : R^2$ to R^2 , as a combination of rotation, dilation and shear, and such that $A(E)$ in a sense close to D . One of the criteria of this proximity may be the minimum value of the area of $|D \text{ setminus } A(E) \cup A(E) \text{ setminus } D|$.
- The final placement of the 3D model is obtained by applying the found affine transformation the vertices of the model.

Implementation of minimizing the described deviation of measures the difference between the area of figures is based on the application package `scipy.optimize` for the Python programming language examples of the use of which is given in [5].

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МАТЕМАТИЧЕСКАЯ МОДЕЛЬ 3D КАРТ И ПРОЕКТИРОВАНИЕ ИНФОРМАЦИОННОЙ СИСТЕМЫ ДЛЯ ИХ УПРАВЛЕНИЯ

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В данной статье описывается процесс создания информационной системы, предназначенной для хранения 3D моделей архитектурных сооружений и комплексов городских микрорайонов, на примере довоенного Сталинграда. Информационная система позволяет управлять файлами моделей, файлами фото и видео материалов, файлами планов и карт городских кварталов, текстовой информацией исторического характера. Также она позволяет «размещать» 3D модели на схемах планов и карт, связывая географические координаты с геометрией модели. Для управления данными информационной системы начата разработка web интерфейса на основе сервера баз данных PostgreSQL и среды разработки Express на базе платформы NodeJS. В статье описана математическая модель «привязки» трехмерных моделей к географическим координатам, связанная с пересчетом координат на основе данных масштаба карты и разрешения графического файла с ее изображением.

Ключевые слова: геометрическая модель; схема базы данных; ER диаграмма; планарное разбиение; PostgreSQL; NodeJS.

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